LOAN AMORTISATION ALGORITHM TYPES, AMORTISATION CHARACTERISTICS AND THEIR FINANCIAL IMPLICATIONS

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In the banking sector, equal loan repayments have been calculated in the same way for centuries, while the nature of the currency in which loans are provided has fundamentally changed. Previously gold standard currencies were used, and when determining the interest rate used in the calculations, no provision was made for the depreciation of money. In this paper, we show how the amortisation method that emerged in the age of gold standard currencies needs to be redefined, due to the risk of modern currencies depreciating with inflation. The new methods should not magnify the impacts of potential changes in interest rates, and should give rise to amortisation characteristics that are more in line with the life cycle in the case of consumer loans, and business activity in the case of corporate loans.

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1. INTRODUCING

With regard to long-term loans, we set out to resolve two tasks: making the repayments more even, and reducing the impact of interest rate change on the repayments. The generally applied solution for the former is to determine nominal repayments of equal amounts. In the case of long maturities, however, this has yielded no optimal solution for reducing the risk of changes in the repayment due to interest rate variation. This is because, where the annuity-based methodology is used, interest rate changes are reflected exponentially in the repayment (see: *Table 1*). With respect to the full term of up to several decades in the case of mortgage loans, no such solution has emerged due to the absence of liquid money-market hedging instruments suitable for fixing the interest rate, and due to the extra costs of interest rate fixing.

Table 1:
Interest rate dependency of annuity loan repayments

Interest (R)	Repayment	Increase	Increase
3%	HUF 55 460		
4%	HUF 60 598	HUF 5 138	8.48%
5%	HUF 65 996	HUF 5 398	8.18%
6%	HUF 71 643	HUF 5 648	7.88%
7%	HUF 77 530	HUF 5 887	7.59%
8%	HUF 83 644	HUF 6 114	7.31%
9%	HUF 89 973	HUF 6 329	7.03%
10%	HUF 96 502	HUF 6 530	6.77%

Note: amount borrowed: HUF 10 000 000, term: 240 months
Source: by author

Recently, where mortgage loans are concerned, the two solutions have been combined on the basis of consumer protection considerations, with interest fixed – as permitted by opportunities in the money market – for several-year cycles (MNB, 2018). This combination is potentially very successful if the beginnings of the interest periods happen to fall at times of "good" low interest rates and expectations of only moderate interest rate changes. The risk, however, is that if the beginning of an interest period falls at a time of very "bad" high interest rates and/or the expectation of a substantial rise in interest, then the increase in the repayments (potentially) causes a shock. The optimal structures described in the following sections aim to correct these typical flaws.

2. PROBLEMS ASSOCIATED WITH ANNUITY LOANS

A popular purpose of financial calculations is determining the annuity-based, fixed-amount repayments on loans. University textbooks usually derive this from the annuity, to arrive at the following result (for consistency with later sections, r is the reference interest rate, m the interest margin of the loan, and let $R = r + m$, while n is the number or repayments, often expressed in time units):

$$Repayment = \frac{Amount\ borrowed}{\frac{1}{R} - \frac{1}{R(1+R)^n}} \tag{1}$$

We prefer not to work with this formula on paper, and indeed there is no need to do so, as financial calculators and computers are preprogrammed with its algorithm. In the past, the interest/term (AF: r, n) pairs were shown in what were known as annuity tables in the textbooks and specialist literature.

The result in (1) can be reached via a shorter route as follows:

– The amount borrowed is precisely equal to the present value of the repayments (X_i) discounted by $R = r + m$, that is

$$Amount\ borrowed = \sum_{i=1}^{n} \frac{X_i}{(1+R)^i}. \quad (2)$$

– For annuity repayment, the repayments are expected to be equal, so
$X_i = X_j = X$. (3)

– Form and sum formula of the general geometric sequence

$$S_n = a_1 \sum_{i=1}^{n} q^{i-1} = a_1 \times \frac{q^n - 1}{q-1} \quad (4)$$

– X can be isolated from formula (2) because of its equivalence with (3) and furthermore, in this case, based on the $a_1 = q = \frac{1}{1+R}$ relationships:

$$Amount\ borrowed = X \times \frac{1}{1+R} \times \frac{\left(\frac{1}{1+R}\right)^n - 1}{\frac{1}{1+R} - 1}, \quad (5)$$

of which:

$$X = \frac{Amount\ borrowed\ \times\ (1+R)\ \times\ \left(\frac{1}{1+R} - 1\right)}{\left(\frac{1}{1+R}\right)^n - 1} = \frac{-\,Amount\ borrowed \times R}{\left(\frac{1}{1+R}\right)^n - 1} \quad (6)$$

– The equivalence of formulas (1) and (6) can be shown with the following rearrangement:

$$\frac{1}{\frac{1}{R} - \frac{1}{R(1+R)^n}} = \frac{-R}{\left(\frac{1}{1+R}\right)^n - 1}.$$

– With both sides rearranged:

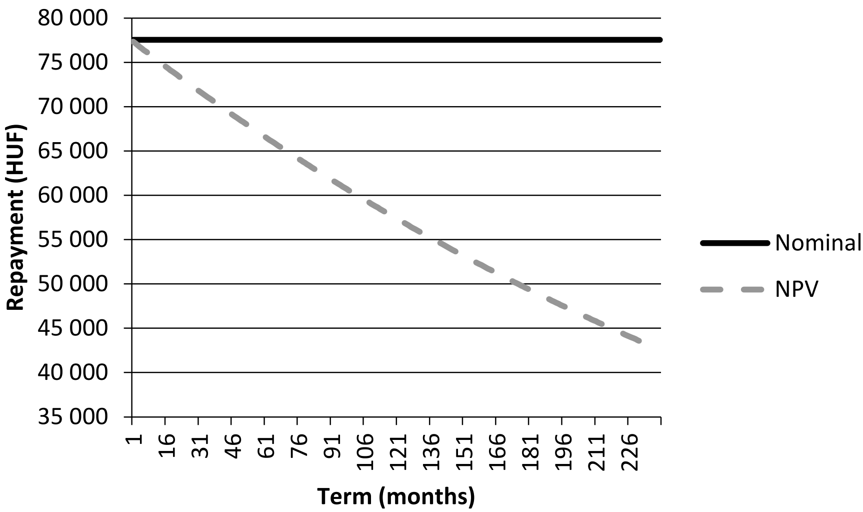
$$\frac{1}{\frac{1}{R} \times \left(1 - \frac{1}{(1+R)^n}\right)} = \frac{R}{-\frac{1}{(1+R)^n} + 1}.$$

– Dividing by the fractionon $\frac{1}{R}$ the left hand side is equivalent to multiplying by R on the right as the reciprocal, and thus the two numerators and the two denominators are the same, so the two sides are equal.

This confirms the equivalence of formulas (1) and (6). This proof was not gratuitous, as it prepares the ground for the later derivations and harmonisation with the results.

The nominal and net present values (NPV) of classic annuity repayments, discounted by *r*, are shown in *Figure 1* in the context of a specific example. The interest rates here, and in what follows, are shown on a p. a. (per annum) basis, and the amount borrowed is denoted by *H*.

Figure 1
Nominal and present value of repayments on a classic annuity loan

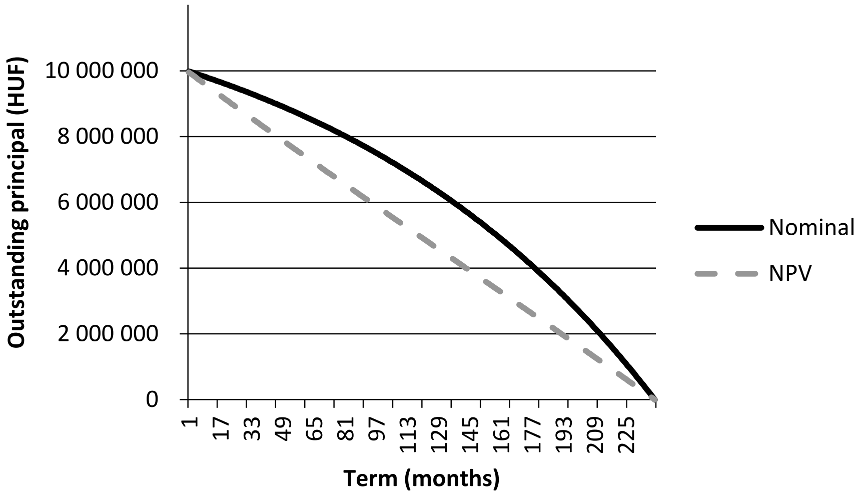


Note: H = HUF 10 000 000, $R = r + m$, $r = 3\%$, $m = 4\%$, $n = 240$ months
Source: by author

As the chart shows, the "price" of having nominally equal repayments is that the initial repayment is relatively high; then as time passes, the monthly repayment burden depreciates with inflation. For mortgage loans, this runs counter to the consumer life cycle, as it overburdens young home buyers in the years following the home purchase; then later, when salaries are likely to stabilise or increase, the repayment burden becomes negligible. The situation is similar for investment loans, as the new investment causes the company's income-generating capacity to increase as time progresses, while the loan burden decreases contrary to this. In other words, here the borrower is overburdened during the initial period, and under-burdened in the closing period.

Due to the lender's risks, we should also look at the value and present value of the outstanding principal debt during the term. Remaining with the previous example, this is shown in chart 2.

Figure 2
Change in nominal and present value of the outstanding principal of a classic annuity loan



Note: H = HUF 10 000 000, $r = 3\%$, $m = 4\%$, $n = 240$ months
Source: by author

As expected, the outstanding principal – due to the initial overburdening – decreases rapidly.

The impact of the interest rate change on the repayment has already been shown in Table 1, so now we will also give it as a function; that is, the total derivative function of (1) with respect to R.

$$X'(R) = -Amount\ borrowed \frac{\frac{-1}{R^2}+\frac{1}{R^2(1+R)^n}+\frac{n}{R(1+R)^{n+1}}}{\left(\frac{1}{R}-\frac{1}{R(1+R)^n}\right)^2} \tag{7}$$

As demonstrated in Table 1, and also observed in the derived function, the effect of the 1 percentage point interest rate increase on the amount of the repayment is exponential, at several times the interest rate increase given a normal level of interest rates.

These problems did not occur in the age of gold standard currencies, because then the repayment burden was the same throughout the term, e.g. 6 pieces of the same gold coin, or banknotes redeemable for gold, every month.

3. OPTIMAL MORTGAGE LOANS WITH A CONSTANT PRESENT VALUE

A prerequisite for the widespread uptake of mortgages is that the reference interest rate should be relatively low (based on general experience, below 10%, because above this the starting monthly repayment is unaffordable for society as a whole), and if possible, interest rates should not be volatile.

This is why, in the past, mortgage loans based on an intermediary currency (e.g. Swiss franc, US dollar) became widespread in several Central and Eastern European and South American countries. With these, the initial repayments were much lower, and the expected amortisation characteristics – being almost constant in terms of their present value – were more in line with the consumer life cycle. Due to the economic crisis, however, a dramatic deterioration in the exchange rates of precisely these currencies, and in the USA the introduction of the right to walk away – as the root cause of the collapse of the mortgage market – decimated the mortgage market. Regarding the change in exchange rates, a practical and theoretical comparison of FCY and HUF-based loan burdens has been performed (*Király–Simonovits*, 2015). However, due to the extreme market impacts and lack of an optimal intermediary currency, it is impossible to build a stable mortgage market on this solution. It should also be mentioned that, aiming for the optimal amortisation characteristics, it would also have been possible to introduce a satisfactory amortisation formula – through the mathematical and optimal mirroring of FCY-based loans – based on the countries' own national currencies. This was recently defined successfully (Kovács–*Pásztor*, 2018). In it, the repayments were determined by formula (8).

$$X_i = \frac{Amount\ borrowed\ \times \left(\frac{1+r+m}{1+m}\right)^i}{\frac{1}{m} - \frac{1}{m(1+m)^n}} \tag{8}$$

The derivation and significance of the formula is presented in the cited study.

We can find the optimal mortgage amortisation process, where it is not the nominal, but the present value of the repayments that is constant, based on the analogue of the derivation encountered at the beginning of the previous section (Kovács–Pásztor, 2018):

– The amount borrowed is precisely equal to the present value of the repayments (X_i) discounted by $r + m$, that is

$$Amount\ borrowed = \sum_{i=1}^{n} \frac{X_i}{(1+r+m)^i}\ . \tag{9}$$

– The equality of the repayments discounted by r is given by the following relationship:

$$X_i = X_0 \times (1 + r)^i \tag{10}$$

where X_0 is the present value of the repayment calculated for the time of borrowing, substituted into the previous formula:

$$Amount\ borrowed = \sum_{i=1}^{n} \frac{X_0(1+r)^i}{(1+r+m)^i}. \tag{11}$$

– Form and sum formula of the general geometric sequence

$$S_n = a_1 \sum_{i=1}^{n} q^{i-1} = a_1 \times \frac{q^n - 1}{q-1}, \tag{12}$$

in formula (11) , $q = a_1 = \frac{1+r}{1+r+m}$ based on these relationships and following the isolation of X_0

$$Amount\ borrowed = X_0 \times \frac{1+r}{1+r+m} \times \frac{\left(\frac{1+r}{1+r+m}\right)^n - 1}{\frac{1+r}{1+r+m} - 1}. \tag{13}$$

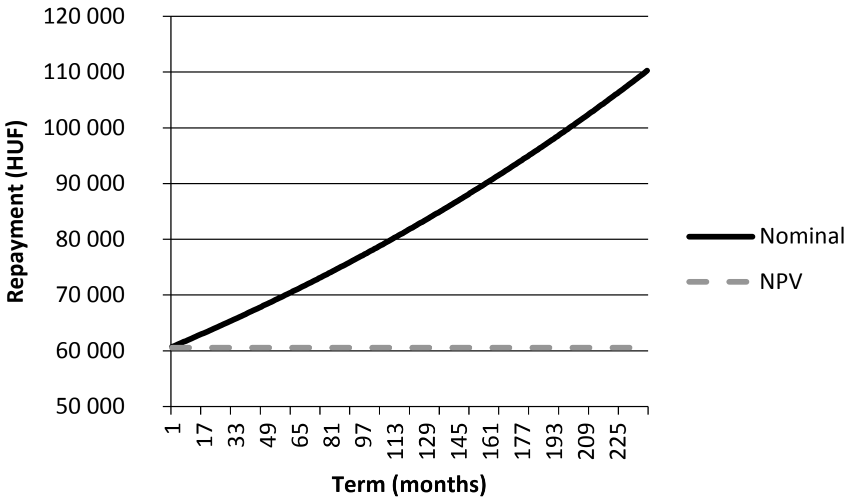
From this, after the restoration of X_i from formula (10) following by simplifications, expressing the i-th repayment:

$$X_i = \frac{Amountborrowed \times (1+r)^i}{\frac{1+r}{1+r+m} \times \frac{\left(\frac{1+r}{1+r+m}\right)^n - 1}{\frac{1+r}{1+r+m} - 1}} = \frac{-Amount\ borrowed\ \times m \times (1+r)^{i-1}}{\left(\frac{1+r}{1+r+m}\right)^n - 1} \tag{14}$$

In other words, with this optimal repayment determination, the present value of every repayment will be the same. Remaining with the same example, the amortisation characteristics, that is, the nominal and present value of the repayments, are shown in *Figure 3*.

Figure 3

Nominal and present value of optimal mortgage loan repayments



Note: H = HUF 10 000 000*, r* = 3%, *m* = 4%, *n* = 240 months
Source: by author

The significance of this result is that the repayment burden of the mortgage loan, provided that the borrower's income is constant in value (e.g. if it continuously rises with the reference interest rate), will remain constant. In other words, it will not be an excessive burden in the initial period (remaining with the same example, HUF 61 000 instead of HUF 78 000), although the repayments will not depreciate with inflation during the closing period. For example, if someone makes a living panning for gold (by which I mean any occupation providing a stable income!), then if they have to pan for one week every month to meet the monthly repayment, then they would have to do it for precisely one week every month throughout the full term of the loan. The interesting theoretical implication of this new approach – which makes sense given narrow limits on changes in income – is that when the amount of the repayment at any given time is linked to prevailing income, changes in tenor are applied instead (*Berlinger–Walter*, 2013).

Another result of the formula is that it means mortgage lending could also be introduced/applied in countries struggling with high interest rates – e.g. those that formerly resorted to the aforementioned foreign-currency mortgage loans – in such a way that the repayments remain affordable throughout the full term of the loan. The initial monthly repayments, e.g. with a 20-year term and 4% interest margin, amount to 0.6% of the amount borrowed, regardless of the reference interest rate.

The change in the level of interest is reflected as a fixed sum in the amount of the repayment in the specific example (see *Table 2*).

Table 2
Interest rate dependency of the first monthly repayment on the optimal mortgage loan

Reference interest rate	1st repayment	Increase (HUF)	Increase (%)
1%	HUF 60 631		
2%	HUF 60 664	HUF 32.94	0.0543%
3%	HUF 60 697	HUF 32.94	0.0543%
4%	HUF 60 730	HUF 32.94	0.0543%
5%	HUF 60 763	HUF 32.95	0.0543%
6%	HUF 60 796	HUF 32.95	0.0542%
7%	HUF 60 829	HUF 32.95	0.0542%
8%	HUF 60 862	HUF 32.96	0.0542%
9%	HUF 60 895	HUF 32.96	0.0542%
10%	HUF 60 928	HUF 32.96	0.0541%

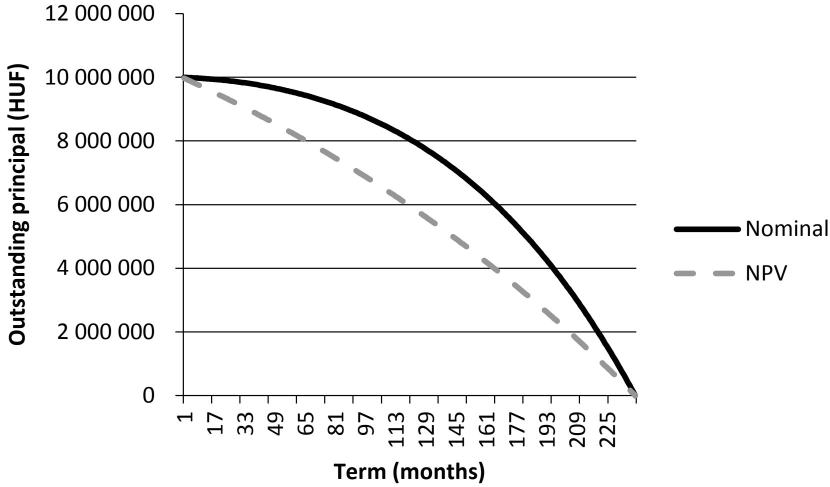
Note: H = HUF 10 000 000, m = 4%, n = 240 months
Source: by author

In other words, with this method, the risk of a change in the interest rate is reflected in a very moderate value increase, which is a complex function of the variables. This function – given the interest rates and terms typical of Hungary – can be approached very well using a linear function. The total derivative of formula (14) with respect to "r" also shows this:

$$X_i{}'(r) = \frac{Hm(1+r)^{i-2}\left[(1-i)(1+r+m)\left(\left(\frac{1+r}{1+r+m}\right)^n - 1\right) + nm\left(\frac{1+r}{1+r+m}\right)^n\right]}{(1+r+m)\left(\left(\frac{1+r}{1+r+m}\right)^n - 1\right)^2} \tag{15}$$

An examination of the outstanding principal cannot be omitted here either. Remaining with the specific example, the nominal and present value of the outstanding principal is shown in *Figure 4*.

Figure 4
Change in nominal and present value of optimal mortgage loan principal



Note: H = HUF 10 000 000, *r* = 3%, *m* = 4%, *n* = 240 months
Source: by author

In other words, the decrease in principal takes place slower than in the case of a classic annuity loan.

4. OPTIMAL INVESTMENT LOAN WITH RISING PRESENT VALUE

Investment loans are also typically long-term facilities, granted by banks to functioning and fundamentally creditworthy companies. Accordingly, for repayment of the loan, the credit institutions not only take into account, and use, the expected income from the new investment, but also the income from other activities of a company that is already trading. An item of trivia evidencing this is that during the grace period following disbursement of the investment loans, when the investment is being implemented and the new unit is not yet generating revenue, credit institutions still request interest payments, at the least. The funds for this can only come from other revenues, or from the investment loan itself.

The first revenues generated by the new investments only start to come in after completion of the investment, and typically increase over time. In other words, the natural requirement for an investment would be a total repayment moratorium (relating to both principal and interest), and after implementation of the investment a steady increase, e.g. by z, in the present value of the repayments. This formula is determined in a similar way to the previous derivation:

– The amount borrowed is precisely equal to the present value of the repayments (X_i) discounted by $r + m$, that is

$$Amount\ borrowed = \sum_{i=1}^{n} \frac{X_i}{(1+r+m)^i}. \tag{16}$$

– The equality of the repayments discounted by $r + z$ is given by the following relationship:

$$X_i = X_0 \times (1 + r + z)^i \tag{17}$$

where X_o is the present value of the repayment calculated for the time of borrowing, substituted into the previous formula:

$$Amount\ borrowed = \sum_{i=1}^{n} \frac{X_0 \times (1 + r + z)^i}{(1 + r + m)^i} \tag{18}$$

– Form and sum formula of the general geometric sequence

$$S_n = a_1 \sum_{i=1}^{n} q^{i-1} = a_1 \times \frac{q^n - 1}{q - 1} \tag{19}$$

in formula (18) $q = a_1 = \dfrac{1 + r + z}{1 + r + m}$ according to these relationships:

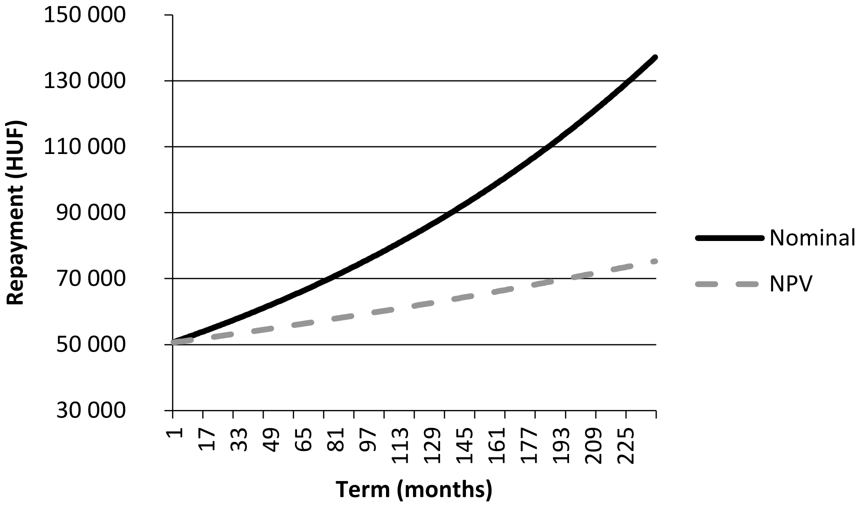
$$Amount\ borrowed = X_0 \times \frac{1+r+z}{1+r+m} \times \frac{\left(\frac{1+r+z}{1+r+m}\right)^n - 1}{\frac{1+r+z}{1+r+m} - 1}. \tag{20}$$

From this, after the restoration of X_i from formula (17) followed by simplifications, expressing the i-th repayment:

$$X_i = \frac{Amount\ borrowed \times (1+r+z)^i}{\frac{1+r+z}{1+r+m} \times \frac{\left(\frac{1+r+z}{1+r+m}\right)^n - 1}{\frac{1+r+z}{1+r+m} - 1}} = \frac{Amount\ borrowed \times (z-m)(1+r+z)^{i-1}}{\left(\frac{1+r+z}{1+r+m}\right)^n - 1}. \tag{21}$$

The nominal and present values of the repayments are shown in *Figure 5*, given a 2% increase in the repayments.

Figure 5
Nominal and present value of optimal investment loan repayments



Note: H = HUF 10 000 000, r = 3%, m = 4%, z = 2%, n = 240 months
Source: by author

In other words, there is a clearly definable investment loan amortisation formula in which the repayments increase as a function of the increase in the r reference interest, m interest margin and z income. The X_0 base repayment does not depend on the reference interest rate! This makes it possible to promote economic development with bank loans provided in the national currency, even in countries struggling with high interest rates.

The dependency of the value of the repayment on changes in the reference interest rate is constant like that of the optimal mortgage loan, due to its similar formula (see *Table 3*).

Table 3
Interest rate dependency of the first monthly repayment on the optimal investment loan

Reference interest rate	repayment	Increase (HUF)	Increase (%)
1%	HUF 50 691		
2%	HUF 50 725	HUF 34.266	0.0676%
3%	HUF 50 760	HUF 34.267	0.0676%
4%	HUF 50 794	HUF 34.268	0.0675%
5%	HUF 50 828	HUF 34.269	0.0675%
6%	HUF 50 862	HUF 34.269	0.0674%
7%	HUF 50 897	HUF 34.270	0.0674%
8%	HUF 50 931	HUF 34.271	0.0673%
9%	HUF 50 965	HUF 34.272	0.0673%
10%	HUF 51 000	HUF 34.272	0.0672%

Note: H = HUF 10 000 000, $r = 3\%$, $m = 4\%$, $z = 2\%$, $n = 240$ months
Source: by author

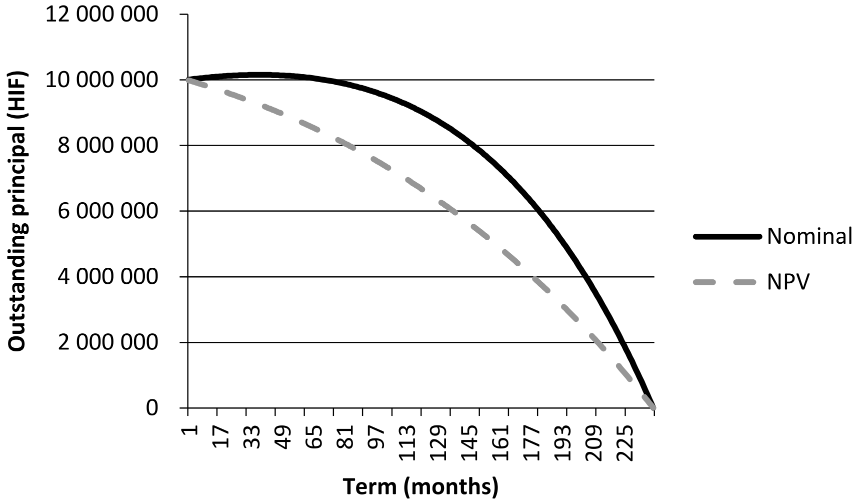
The total derived function of formula (21) with respect to r:

$$X_i{}'(r) = \frac{H(m-z)(1+r+z)^{i-2}\left[(1-i)(1+r+m)\left(\left(\frac{1+r+z}{1+r+m}\right)^n - 1\right) + n(m-z)\left(\frac{1+r+z}{1+r+m}\right)^n\right]}{(1+r+m)\left(\left(\frac{1+r+z}{1+r+m}\right)^n - 1\right)^2} . \qquad (22)$$

As Table 3 also shows, the derived function can be approached well with the linear curve, given the usual level of the reference interest rate.

The nominal and present values of the outstanding principal are shown in *Figure 6*.

Figure 6
Change in nominal and net present value of optimal investment loan principal



Note: H = HUF 10 000 000, *r* = 3%, *m* = 4%, *z* = 2%, *n* = 240 months
Source: by author

It is clear that the decrease in principal takes place more slowly than before. The repayment burdens, however, only become greater when the upturn in revenues is also taking place. The "price" of this is that the decrease in outstanding principal – possibly following a temporary increase – is concentrated in the closing phase.

A clear advantage of the new, optimal method is that the amortisation characteristics are much more closely aligned with the projected income from the new investments, and the dependency of repayments on the reference interest rate and reference interest rate changes is low. These characteristics can facilitate globally predictable and continuous economic growth given the appropriate activity on the part of credit institutions.

5. POTENTIAL SOCIAL-POLICY IMPLICATIONS

It is worth weighing up the pros and cons of loan facilities amortised using the optimal formula. Their advantage is that they make it possible to determine a payment burden that is either constant throughout the term, or aligned with projected revenue growth. If the interest rate is fixed until maturity, then the regular repayment obligation can also be determined in advance for the full term. If the loan is provided on a variable-interest basis, then the mid-term changes in interest are reflected in the repayments in a way that is effectively linear and matches the extent of the change in interest.

It could be seen as a disadvantage that, unlike the facilities we have been accustomed to, the repayments do not depreciate with inflation. In the case of a variable interest rate, the repayments are only known for a given period (this might be the next repayment, but may also be fixed for several repayment cycles), so the precise extent of the next repayment carries some uncertainty if the reference interest rate will change in the meantime. From the banks' perspective, the duration of the loan receivable is longer, which is a disadvantage if payment discipline is bad, but an advantage in the case of good payment discipline. Moreover, not even the optimal methods are capable of managing the drop in income that results from the loss of a job, the freezing of income levels during an economic crisis, extreme volatility in individual property markets, etc. Here is should be mentioned that, for general purposes, the statutory frameworks for mortgage lending should be aligned with the new structures; for example, it makes no sense to compare today's income with the repayments due in 20 years' time.

In summary, the benefits are desirable from a consumer protection standpoint, while the drawbacks are typically less disadvantageous than those of the customary annuity structures.

The study has shown that, irrespectively of the reference interest rate, the initial repayment of a HUF 10 million mortgage loan with a 20-year term and 4% margin is HUF 60 000. Meanwhile, rent is around 0.8-1% of the property's value. In other words, given a mortgage loan structure that affords sufficient lender protection, even for a property purchase with no upfront payment, the monthly repayment remains less than the rent would be. The latter statement will remain true in the next two decades if property prices, rents and incomes, and thus the repayments, move together (e.g. if they follow inflation). This optimal mortgage structure could also be used globally to resolve the acquisition of property among the Earth's population; because, as we have shown, the cost of acquiring the property remains below that of the alternative, renting. This is the only chance for the poor, aspirational sections of the population to fund their own home acquisition.

The use of a maximum 20-year term is also ethical, as it provides a realistic opportunity for the population, given the average time spent in work (40-50 years), to accumulate other wealth in addition to their home. This can be regarded as a financial prerequisite for middle class growth, because if "we don't live to eat", then we should not work just to have somewhere to live either.

6. SUMMARY

A fundamental problem with classic annuity loan repayments is, firstly, that the loan amortisation characteristics are not aligned with the consumer life cycle in the case of consumer mortgage loans, or with a business plan that is founded on growing revenues in the case of investments. Secondly, interest rate levels, and their volatility, are reflected exponentially in the extent and volatility of the repayments. These problematic factors are eliminated by the optimal loan amortisation formulae (as a comparison of the tables in the Appendixes shows!)

The formula for the *i*-th repayment of the optimal mortgage loan (*r* – reference interest rate, *m* – interest margin, *n* – number of repayments, *H* amount borrowed):

$$X_i = \frac{-Hm(1+r)^{i-1}}{\left(\frac{1+r}{1+r+m}\right)^n - 1}$$

The formula of the *i*-th repayment of the optimal investment loan (*r* – reference interest rate, *m* – interest margin, *z* – increase in repayment, *n* – number of repayments, *H* amount borrowed):

$$X_i = \frac{H(z-m)(1+r+z)^{i-1}}{\left(\frac{1+r+z}{1+r+m}\right)^n - 1}$$

By introducing the new, optimal structure, credit institutions can enter new markets (those struggling with high interest rates). It is sufficient to provide the funds in the national currency; the use of a variable interest rate does not require costly long-term, fixed-interest funds, so overall the loans can be covered relatively cheaply. With the optimal structures, because the repayments are fixed at present value, the duration of the loan portfolios increases; in other words, their existing liquidity can be placed for a longer period on average.

We arrived at the optimal loan amortisation formulas with novel and spectacular mathematical derivations. For customers, use of the optimal structures results in lower initial repayments, but the constancy of the present value of the repayments, and their pre-planned nominal increase, is aligned with the natural consumer need associated with retail mortgage and corporate investment loans. The price of this is that the repayments change constantly (e.g. monthly, quarterly or every six months), which necessitates some IT development on the part of the banks, and more care on the part of customers.

In the case of the optimal loan amortisation formulas, the impact on repayments of the rate and volatility of interest is moderate and almost linear.

For consumers, the optimal loan structures provide a genuine and strong alternative to renting, so globally their use can be used to help resolve humanity's housing issues, while in the corporate sphere the alignment of loan amortisation characteristics with the projected revenue from new investments offers new solutions for sustainable economic growth based on the provision of credit.

APPENDIX

1/A. Amortisation schedule

Classic annuity loan

Amount borrowed	Term (years)	Reference interest rate	Interest margin
10 000 000	**20**	**3%**	**4%**

One repayment per annum!

Year	Annual repayment	NPV Annual repayment	Interest	Principal	Principal remaining	NPVPrincipal remaining
1	**943 929**	916 436	700 000	243 929	9 756 071	9 471 913
2	**943 929**	889 744	682 925	261 004	9 495 066	8 950 011
3	**943 929**	863 829	664 655	279 275	9 215 792	8 433 755
4	**943 929**	838 669	645 105	298 824	8 916 968	7 922 611
5	**943 929**	814 242	624 188	319 741	8 597 227	7 416 043
6	**943 929**	790 526	601 806	342 123	8 255 103	6 913 519
7	**943 929**	767 501	577 857	366 072	7 889 031	6 414 504
8	**943 929**	745 146	552 232	391 697	7 497 334	5 918 465
9	**943 929**	723 443	524 813	419 116	7 078 218	5 424 865
10	**943 929**	702 372	495 475	448 454	6 629 764	4 933 167
11	**943 929**	681 915	464 083	479 846	6 149 918	4 442 832
12	**943 929**	662 053	430 494	513 435	5 636 483	3 953 316
13	**943 929**	642 770	394 554	549 375	5 087 108	3 464 073
14	**943 929**	624 048	356 098	587 832	4 499 276	2 974 552
15	**943 929**	605 872	314 949	628 980	3 870 296	2 484 196
16	**943 929**	588 226	270 921	673 009	3 197 288	1 992 444
17	**943 929**	571 093	223 810	720 119	2 477 169	1 498 728
18	**943 929**	554 459	173 402	770 527	1 706 641	1 002 472
19	**943 929**	538 310	119 465	824 464	882 177	503 093
20	**943 929**	522 631	61 752	882 177	0	0

1/B. Amortisation schedule

Optimal mortgage loan

Amount borrowed	Term (years)	Reference interest rate	Interest margin
10 000 000	**20**	**3%**	**4%**

One repayment per annum!

Year	Annual repayment	NPV Annual repayment	Interest	Principal	Principal remaining	NPV Principal remaining
1	750 094	**728 247**	700 000	50 094	9 949 906	9 660 103
2	772 597	**728 247**	696 493	76 103	9 873 803	9 307 006
3	795 775	**728 247**	691 166	104 608	9 769 194	8 940 197
4	819 648	**728 247**	683 844	135 804	9 633 390	8 559 142
5	844 237	**728 247**	674 337	169 900	9 463 490	8 163 289
6	869 564	**728 247**	662 444	207 120	9 256 370	7 752 064
7	895 651	**728 247**	647 946	247 706	9 008 664	7 324 868
8	922 521	**728 247**	630 606	291 914	8 716 750	6 881 083
9	950 197	**728 247**	610 172	340 024	8 376 726	6 420 063
10	978 702	**728 247**	586 371	392 332	7 984 394	5 941 139
11	1 008 064	**728 247**	558 908	449 156	7 535 238	5 443 616
12	1 038 305	**728 247**	527 467	510 839	7 024 399	4 926 772
13	1 069 455	**728 247**	491 708	577 747	6 446 653	4 389 857
14	1 101 538	**728 247**	451 266	650 273	5 796 380	3 832 090
15	1 134 584	**728 247**	405 747	728 838	5 067 542	3 252 663
16	1 168 622	**728 247**	354 728	813 894	4 253 648	2 650 733
17	1 203 681	**728 247**	297 755	905 925	3 347 723	2 025 427
18	1 239 791	**728 247**	234 341	1 005 450	2 342 273	1 375 838
19	1 276 985	**728 247**	163 959	1 113 026	1 229 247	701 022
20	1 315 294	**728 247**	86 047	1 229 247	0	0

1/C. Amortisation schedule

Optimal investment loan

Amount borrowed	**Term (years)**	**Reference interest rate**	**Interest margin**	**Increase (z)**
10 000 000	**20**	**3%**	**4%**	**2%**

One repayment per annum!

Year	Annual repayment	NPV Annual repayment	Interest	Principal	Principal remaining	NPV Principal remaining
1	636 259	**617 727**	700 000	-63 741	10 063 741	9 770 622
2	668 072	**629 722**	704 462	-36 390	10 100 131	9 520 342
3	701 475	**641 949**	707 009	- 5 534	10 105 665	9 248 115
4	736 549	**654 414**	707 397	29 153	10 076 512	8 952 851
5	773 377	**667 122**	705 356	68 021	10 008 491	8 633 413
6	812 045	**680 075**	700 594	111 451	9 897 040	8 288 615
7	852 648	**693 281**	692 793	159 855	9 737 185	7 917 223
8	895 280	**706 742**	681 603	213 677	9 523 508	7 517 945
9	940 044	**720 466**	666 646	273 399	9 250 110	7 089 439
10	987 046	**734 455**	647 508	339 539	8 910 571	6 630 302
11	1 036 399	**748 716**	623 740	412 659	8 497 912	6 139 073
12	1 088 219	**763 255**	594 854	493 365	8 004 547	5 614 229
13	1 142 630	**778 075**	560 318	582 311	7 422 236	5 054 182
14	1 199 761	**793 183**	519 557	680 205	6 742 032	4 457 277
15	1 259 749	**808 585**	471 942	787 807	5 954 225	3 821 790
16	1 322 737	**824 286**	416 796	905 941	5 048 284	3 145 924
17	1 388 873	**840 291**	353 380	1 035 493	4 012 790	2 427 804
18	1 458 317	**856 608**	280 895	1 177 422	2 835 369	1 665 480
19	1 531 233	**873 241**	198 476	1 332 757	1 502 612	856 918
20	1 607 795	**890 197**	105 183	1 502 612	0	0

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